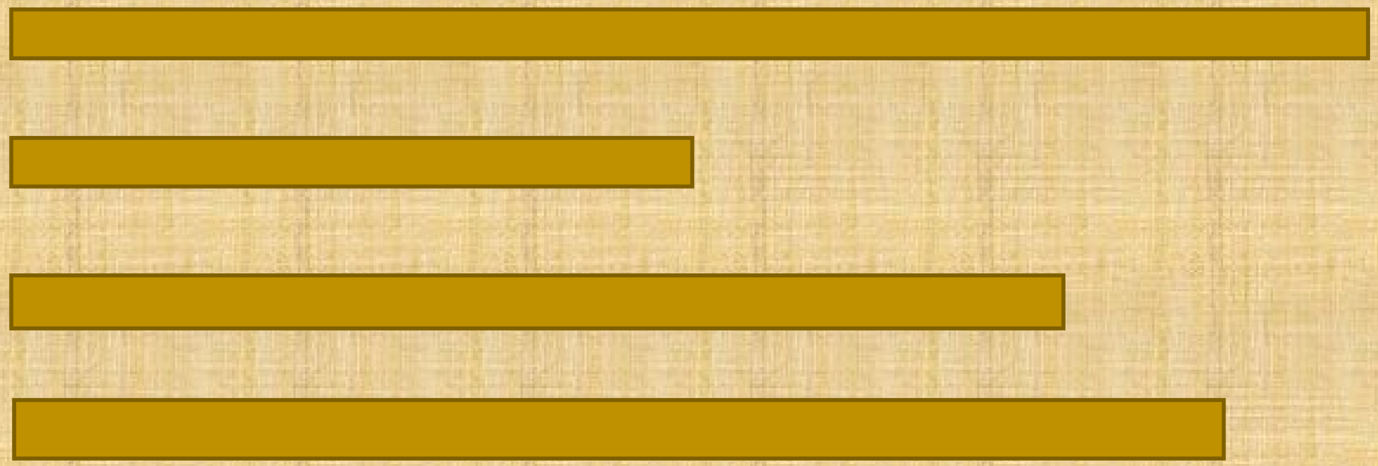
Ancient Egyptian Mathematics Lesson 3: Approximations and Operations

Egyptian Faience wꜣḏ William the Hippopotamus from the Met

Approximation

Consider the length measurement system given below where each part corresponds to a unit length:

Estimate the following lengths:



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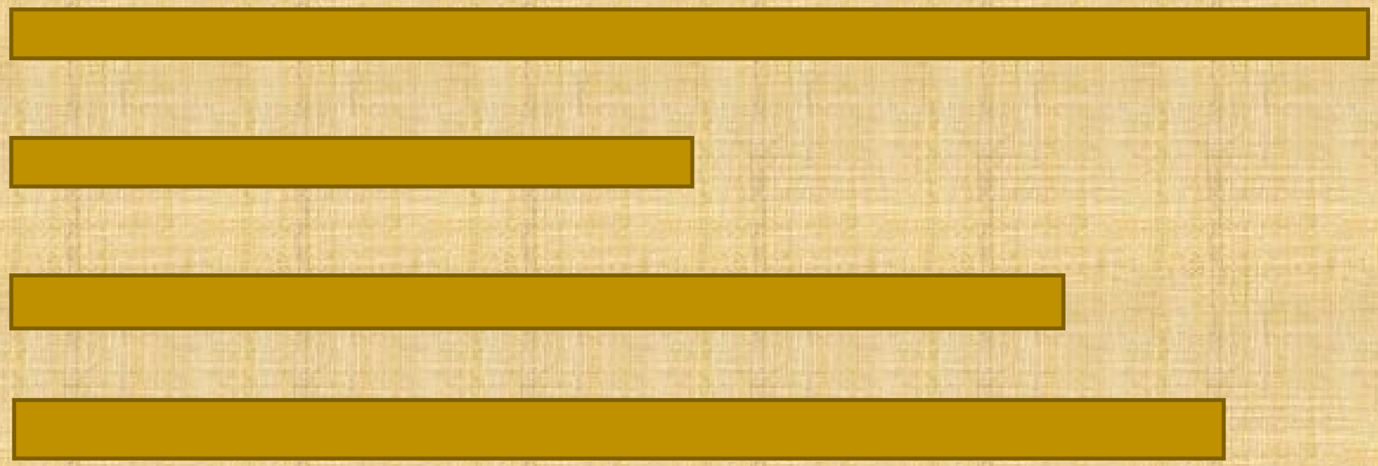

Approximation

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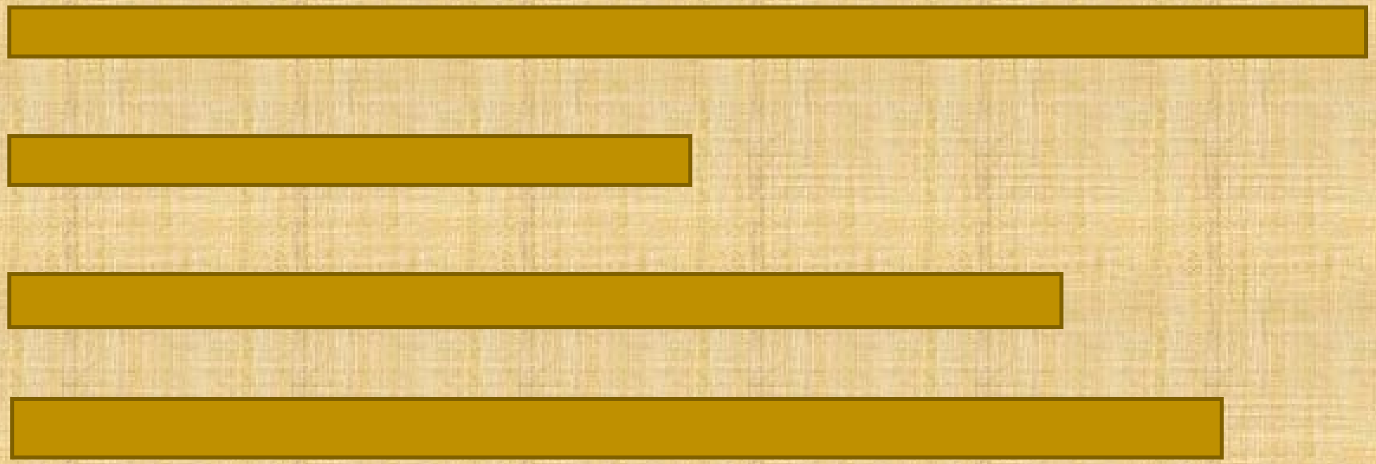
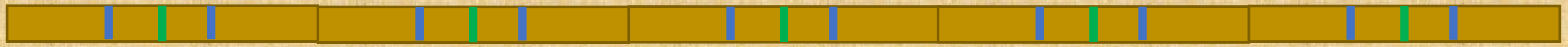


A natural way of approximating a length is to introduce the same concept as pesu from our discussion of bread loaves: the *number of constituents within a single unit length*

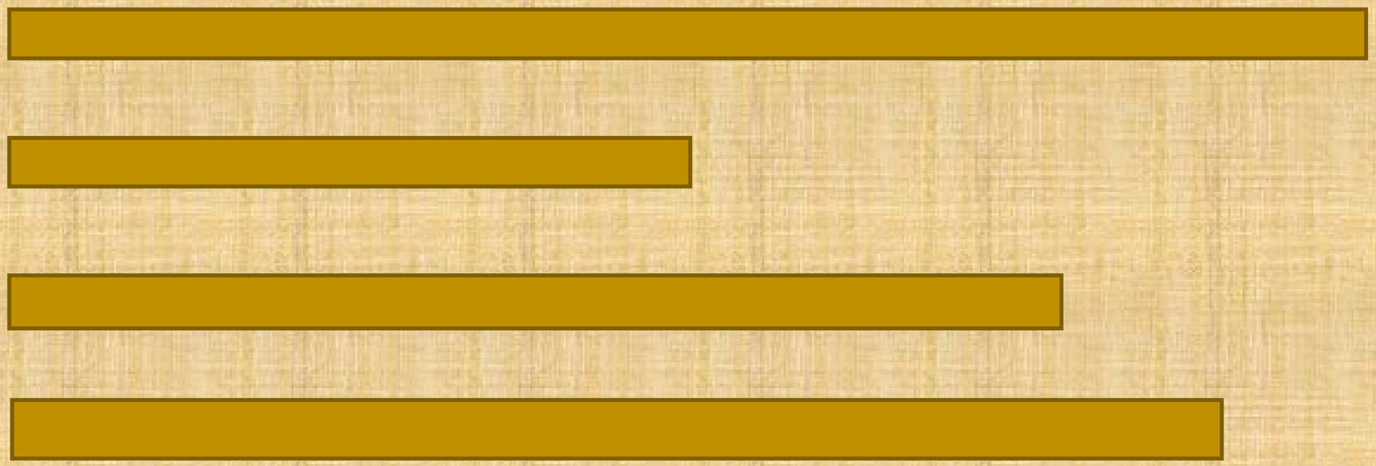
Approximation

Consider the length measurement system given below

where each part corresponds to a unit length:



Estimate the following lengths:



3

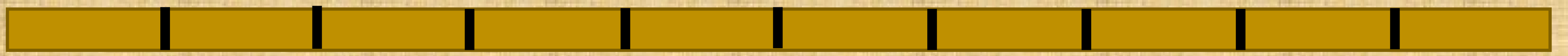
1 $\overline{2}$

2 $\overline{3}$

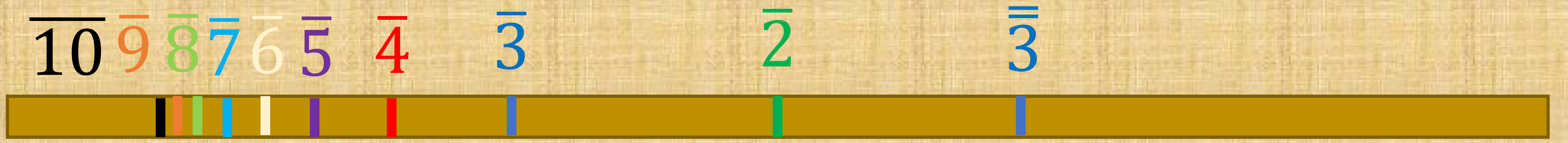
2 $\overline{\overline{3}}$

Egyptian Fractions

Our Decimal System:



Egyptian System:



The significant difference is that the Egyptian system is based off pesu, meaning the length of a single sublength of a *higher pesu* necessarily becomes *smaller*. The decimal system has constant spacing: 0.2 - 0.1 = 0.3 - 0.2 = 0.4 - 0.3 = … = 1 – 0.9 but the Egyptian system has $\overline{2} - \overline{3} > \overline{3} - \overline{4} > \overline{4} - \overline{5}\ldots$

This has implications for the accuracy of measurements

Egyptian Fractions

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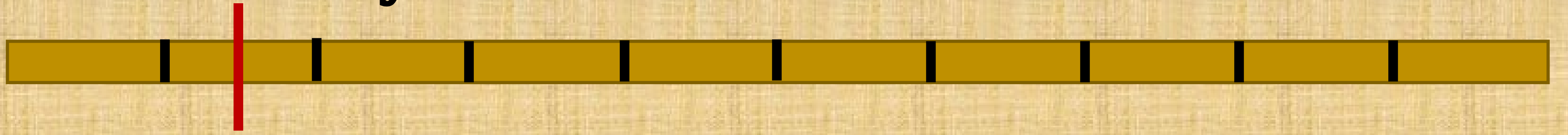
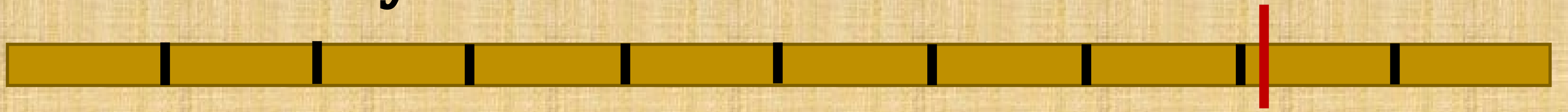


Egyptian System:

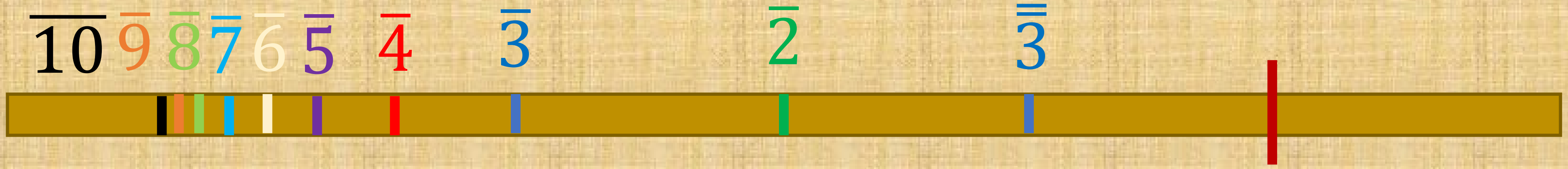
The accuracy of measurements is determined by the spacing since intermediary results are *rounded down to the greatest portion* they contain. The spacing is constant in the decimal system meaning that for the marked measurement, you would be forced to go down to 0.1 but clearly $\overline{7}$ is a more accurate approximation. *The Egyptian fraction approximation tends to be more* accurate *than the decimal system approximation* for small numbers.

Egyptian Fractions

Our Decimal System:



Egyptian System:



But for this marking, the decimal approximation of 0.8 is more accurate than $\overline{\overline{3}}$.

The Egyptian fraction approximation tends to be less accurate than the decimal system approximation for larger numbers.

Ostraca Tablets: Why would it be helpful to have increased measurement accuracy for small numbers near 0? Is it worth the poor accuracy near 1?

Working with $\overline{2}$ and $\overline{\overline{3}}$

Analogous to how multiplication by 2 is well-defined in Egyptian, we also have natural ways to multiply by $\overline{2}$ and $\overline{\overline{3}}$

Multiplication by $\overline{2}$

If n is even, n × $\overline{2}$ is the number that when collected with itself gives n. Example: 6 × $\overline{2} = 3$ since 2 × $3 = 3 + 3 = 6$.

If n is odd, n × $\overline{2}$ is the sum of $\overline{2}$ and the number that when collected with itself gives n – 1. Example: 7 × $\overline{2} = 3\,\overline{2}$ since 2 × $3 = 3 + 3 = 6$ and $6 + 1 = 7$.

Ostraca Practice! 8 × $\overline{2}$, 13 × $\overline{2}$, 2 × $\overline{2}$

Working with $\bar{2}$ and $\bar{\bar{3}}$

Analogous to how multiplication by 2 is well-defined in

Egyptian, we also have natural ways to multiply by $\bar{2}$ and $\bar{\bar{3}}$

Multiplication by $\bar{\bar{3}}$

If n is a multiple of 3, n × $\bar{\bar{3}}$ is given as the number m collected with itself where m is the number than when collected with itself and then collected with m again yields n. Example: 9 × $\bar{\bar{3}} = 6$ since we find that m = 3 because $2 \times 3 = 3 + 3 = 6$ and then $6 + 3 = 9 = \mathrm{n}$. Then 9 × $\bar{\bar{3}}$ = 2 × $\mathrm{m} = m + m = 3 + 3 = 6$

Working with $\bar{2}$ and $\bar{\bar{3}}$

Analogous to how multiplication by 2 is well-defined in

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Multiplication by $\bar{\bar{3}}$

If n is one more than a multiple of 3 (congruent to 1 mod 3), n × $\bar{\bar{3}}$ is given as the number m collected with itself collected with $\bar{\bar{3}}$ where m is the number than when collected with itself and then collected with m again yields n-1.

Example: $10 \times \bar{\bar{3}} = 6\,\bar{\bar{3}}$ since we find that m = 3 because $2 \times 3 = 3 + 3 = 6$ and then $6 + 3 = 9$ and $9 + 1 = 10 = n.$

Then $10 \times \bar{\bar{3}} = 2 \times \mathrm{m} + \bar{\bar{3}} = m + m + \bar{\bar{3}} = 3 + 3 + \bar{\bar{3}} = 6\,\bar{\bar{3}}$

Working with $\bar{2}$ and $\bar{\bar{3}}$

Analogous to how multiplication by 2 is well-defined in

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Multiplication by $\bar{\bar{3}}$

If n is two more than a multiple of 3 (congruent to 2 mod 3), n × $\bar{\bar{3}}$ is given as the number m collected with itself collected with 1 $\bar{3}$ where m is the number than when collected with itself and then collected with m again yields n-2.

Example: 11 × $\bar{\bar{3}} = 7\,\bar{3}$ since we find that m = 3 because $2 \times 3 = 3 + 3 = 6$ and then $6 + 3 = 9$ and $9 + 2 = 11 = n$.

Then 11 × $\bar{\bar{3}}$ = 2 × $\mathrm{m} + 1\,\bar{3} = m + m + 1\,\bar{3} = 3 + 3 + 1\,\bar{3} = 7\,\bar{3}$

Tabular Multiplication and Division

Multiplication using table: "multiply 12 by 4"

12		
1	4	
2	8	
4	16	✓ Use
8	32	✓ Use
	48	

Division using table for whole numbers: "divide 12 by 4"

	12	
1	4	✓ Use
2	8	✓ Use
4	16	
8	32	
3		

Tabular Multiplication and Division

Problem: "divide 81 by 6"

	81	
1	6	✓ Use
2	12	
4	24	✓ Use
8	48	✓ Use
13	**78**	

	81	
$\overline{2}$	3	✓ Use
1	6	✓ Use
2	12	
4	24	✓ Use
8	48	✓ Use

13 $\overline{2}$

Tabular Multiplication and Division

Problem: "divide 18 by 8"

	18	
$\overline{4}$	2	✓ Use
$\overline{2}$	4	
1	8	
2	16	✓ Use

2 $\overline{4}$

Problem: "divide 19 by 3"

	19	
$\overline{4}$	$\overline{2}$ $\overline{4}$	✓ Use
$\overline{2}$	1 $\overline{2}$	
1	3	
2	6	✓ Use
4	12	✓ Use

18 $\overline{2}$ $\overline{4}$

	19	
$\overline{16}$	$\overline{8}$ $\overline{16}$	✓ Use
$\overline{8}$	$\overline{4}$ $\overline{8}$	
$\overline{4}$	$\overline{2}$ $\overline{4}$	✓ Use
$\overline{2}$	1 $\overline{2}$	
1	3	
2	6	✓ Use
4	12	✓ Use

18 $\overline{2}$ $\overline{4}$ $\overline{8}$ $\overline{16}$

Tabular Multiplication and Division

Problem: “divide 19 by 3”

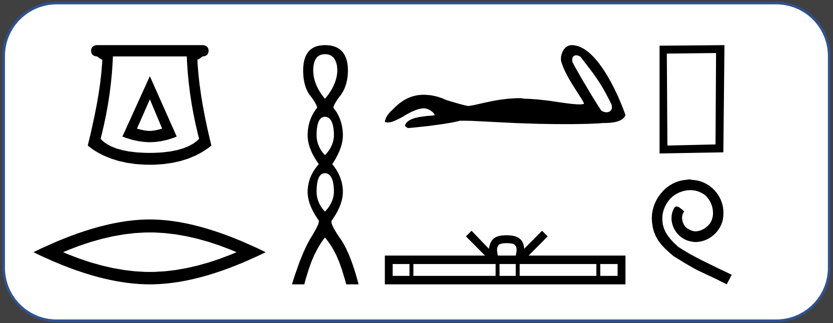
	19	
$\overline{3}$	1	✓ Use
1	3	
2	6	✓ Use
4	12	✓ Use

$6\ \overline{3}$

Problem: “divide 20 by 3”

	20	
$\overline{\overline{3}}$	2	✓ Use
1	3	
2	6	✓ Use
4	12	✓ Use

$6\ \overline{\overline{3}}$



grḥ pw

"It is the end"

šms hrw nfr

"Follow a good day"